Discussion of:
"Sequential Quasi-Monte-Carlo Sampling"
by Mathieu Gerber and Nicolas Chopin

Zhijian He
Department of Mathematics
Tsinghua University

Art B. Owen*
Department of Statistics
Stanford University

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We congratulate Gerber and Chopin for a very interesting paper with much promise for applications. SQMC is similar to array-RQMC (L'Ecuyer et al., 2008), in using T sets of N points in $[0,1]^d$ instead of one point set of dimension dT. The trick is in connecting the N output states generated by time t to N QMC vectors used to generate step $t+1$. The Hilbert curve yields the missing 'sorting hat' making it possible to prove consistency and even a convergence rate, despite the lack of smoothness in matching output to input.

We looked (He and Owen, 2014) into the related, much simpler problem of piping a one dimensional RQMC point set through the Hilbert function in order to get a d dimensional point set. We showed that the mean squared error rate is $O(n^{-1-2/d})$ for Lipshitz continuous integrands of dimension $d \geq 3$. Dimension has an adverse effect and we predict the same for SQMC. Although that rate is unpleasant for large d, it is known to be best possible (Novak, 1988). QMC often involves tricks to reduce effective dimension (Caflisch et al., 1997). Some of those methods might pay off for SQMC.

We would like to note one escape route from having to write the simulation as an explicit function of uniform variables. Usually the problem is how to handle acceptance-rejection sampling. That can be done with uniform variables but requires an indefinitely large number of them. One can use RQMC for the first few attempts (maybe just one) and then paste in independent uniform random numbers for any needed followup sampling. Hörmann et al. (2004) include many acceptance-rejection schemes with acceptance rate above $1-\epsilon$ for any $\epsilon > 0$. That ϵ might spoil the convergence rate but still leave us with a big reduction in the error constant.

We were concerned that the choice of ψ could be critical. Suppose for instance that the outputs at step t have much heavier distribution than logistic. Then the Hilbert curve will only encounter points in or near the 2^d corners of

*Corresponding author: owen@stat.stanford.edu

$[0,1]^d$. Conversely, if those outputs have a much smaller variance than the logistic the curve will find those points mostly near the center $(1/2, 1/2, \ldots, 1/2)$ of the cube. We have observed just such anomalies in a two dimensional stochastic volatility example, but interestingly, the anomalies had no material effect on the variance of our estimates.

References

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