Stat 315c: Introduction

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Stanford Statistics

Stat 315c "Analysis of Transposable Data"

Usual Statistics Setup

- there's Y (we'll predict it)
- and there's $X_1, \ldots, X_d$ (to predict from)
- and n IID copies of (X, Y) to infer with

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- $d+1$ named columns (variables)
- and n anonymous exchangeable rows
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This course

- Both rows and columns are named
- We learn about the cols using rows as obs, and conversely
- n and d may both be large

Problem domains

Two mode examples

- Movies $\times$ Raters $\longrightarrow$ Ratings
- Terms $\times$ Documents $\longrightarrow$ Counts
- Genes $\times$ Experiments $\longrightarrow$ Expression level
- IP-address $\times$ Books $\longrightarrow$ Purchases
- Questions $\times$ Test takers $\longrightarrow$ Grade

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Single mode examples (Rows $\times$ Cols are the same entities)

- Actors $\times$ Actors $\longrightarrow$ co-appearance
- Articles $\times$ Articles $\longrightarrow$ co-citation
- Web pages $\times$ Web pages $\longrightarrow$ hyper-links

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Higher dimensional layouts

- genes $\times$ conditions $\times$ tissues

About the course

History

- Response to common thread in lots of problems
- Began as seminar in spring 2000
- Guest speakers from Netflix
- and biomedical informatics
- and statistics

Goals

- Look at existing methods
- Look for remaining holes

Materials

- Articles online
- There's no book

Data types

Dyadic data

For $X \in \mathcal{X}$ (eg actors) and $Y \in \mathcal{Y}$ (eg movies)
Record pairs

$$(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), \ldots (X_N, Y_N)$$

Actor X_i was in movie Y_i
$N \ll |\mathcal{X}| \times |\mathcal{Y}|$ so the full matrix would be very sparse

So it is "variables and cases as usual", after all

- Variable 1 = actor, Variable 2 = movie
- maybe Variable 3 = box office
- N = # pairs $\to \infty$, with d = 2 or 3

(Well almost)

Back to anonymous rows, but with

A special kind of random variable

- Categorical with many levels, e.g.:
  1. Phone number
  2. IP address
  3. Actor
  4. Query string
- Number of levels grows with N
- There may be many unseen levels

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Different from classical categorical variables, e.g.:

- Binary variables, or,
- Setosa vs Verginica vs Versicolor, etc.

Non-dyadic examples

Dense data

- for microarrays we have all genes in all experiments apart from missing values
- for dyadic case, it's mostly missing apart from a few observed values

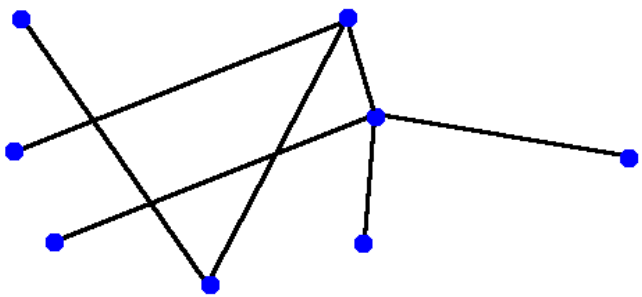
Triadic data

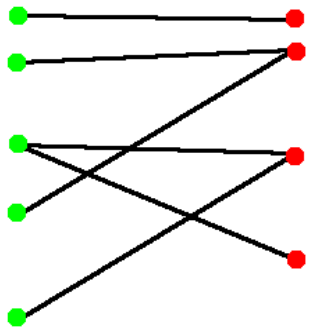
- Actors, Directors, and Year
- Genes, Conditions, Tissues

Graphs

Transposable data often have a graph representation.

- Edges can be directed or undirected
- Bipartite graphs for data of two modes (eg actors and films)





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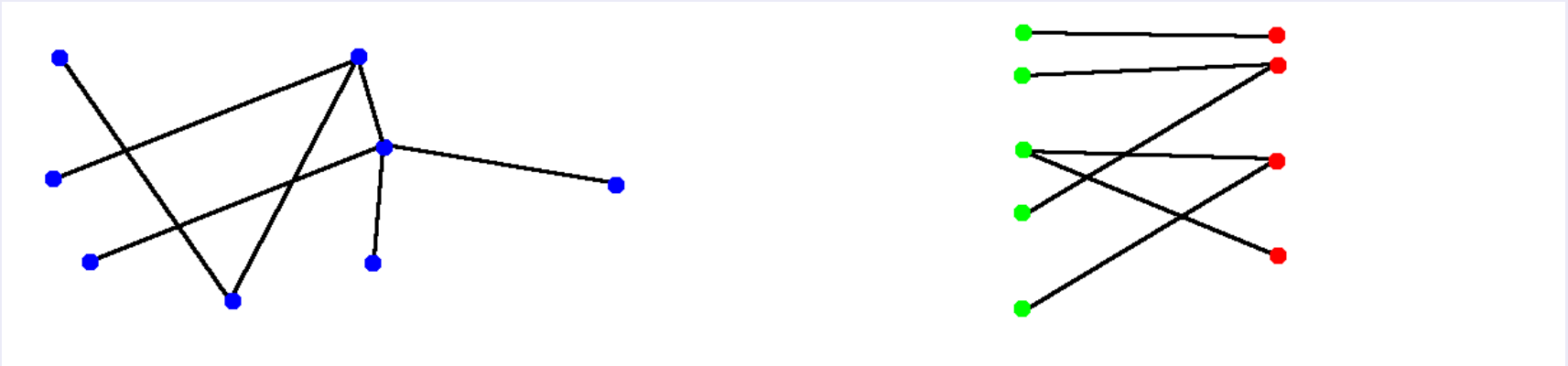

- Edges can have weights (more generally feature vectors)
- Nodes can have features
- Hypergraphs for triadic data
- Generalize ad infinitum (but then we break the graph paradigm)

Methods

Methods for these problems are of several (overlapping) types

- Classical
  1. ANOVA
  2. Correspondence analysis
  3. Rasch model

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- Unsupervised learning
  1. Clustering (group the rows or the columns)
  2. Biclustering (jointly group the rows and the columns)
  3. Spectral clustering
  4. Independent components analysis

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- Matrix approximation
  1. Singular value decomposition
  2. Nonnegative decomposition
  3. Semi-Discrete decomposition

But wait there's more

Some more ideas, not yet forced into a category

- PageRank, TrustRank, Hubs and Authorities
- Smoothing on graphs
- Subsampling matrices
- Recommender engines
- Archetypal analysis
- Latent Dirichlet Allocation
- Compositional data
- Canonical correlation and generalizations
- Head versus long tail

Problems and tasks

We'd like to

- Predict missing labels (eg spam)
- Find anomalies (eg unusual credit card patterns)
- Decide where to get labels
- Group rows/columns/both
- Reduce dimension
- Predict missing links

Goals

- Find common structures in these problem
- Learn some specific methods
- Learn to compare|mix|hybridize methods
- Move from "could to" to "should do"
- Spot research opportunities

High level view

Approaches include

- Principled Bayesian methods
- Ad hoc but very fast algorithms
- Moments
- Maximum likelihood

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Persistent issues

- What happens to the bootstrap and cross-validation?
- How should we window data arriving in time?
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- Right now there are disconnected islands

Not a usual course

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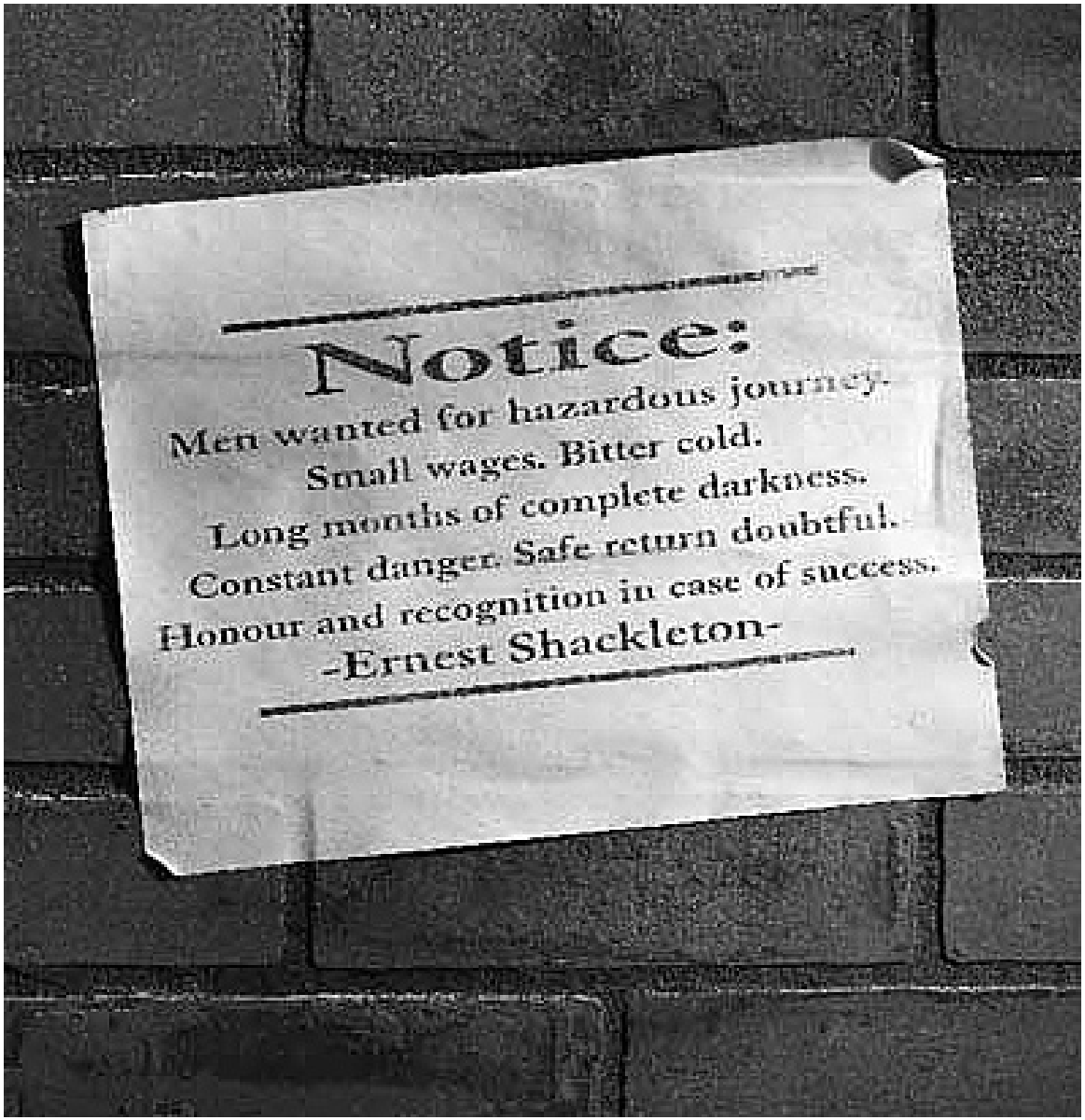

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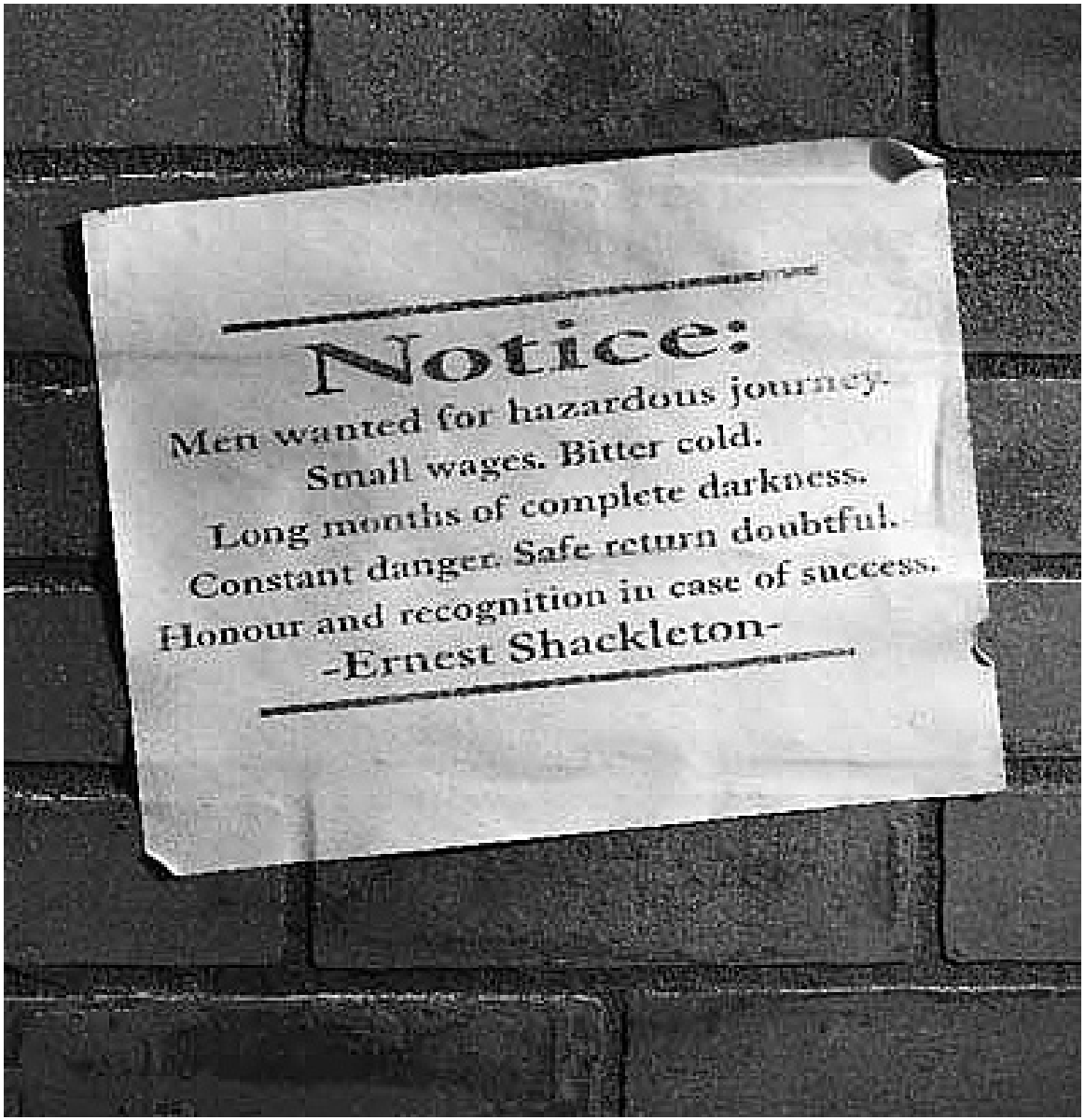

Then

- It was 1914
- 5000 people applied

Now

- Men and women wanted
- It won't be cold

Some results

From 07/08

- New cross-validation method for (Perry and O.)
- New bootstrap for non-IID data (O.)
- Two papers on spectral clustering (Salzman)