

Statistics 200

Problem set 0
0% of your grade

Basic probability

Abstract

Statistics 200 has a prerequisite: Statistics 116. That is a course where you learn basic probability, about distributions, moments, independence, and so on. Ideally you know that material cold. Quite possibly, you took it a while ago and have become rusty. If so, go over your old notes or text. We will review it in the first week and maybe pull in some touch ups as the course goes on. If most of the material below is unfamiliar to you, then you should postpone taking Statistics 200.

If you need to add assumptions to answer these questions, then do it. Some of the questions require extra assumptions or at least require them to be precise. These are not exam questions; they are posed more loosely and they are about whether you know how and where to use probability ideas along with some that are just vocabulary.

You should be comfortably able to get most of these questions. A few of them are tricky.

The questions

1. The planet Tralfamadore has years with 500 days. There are 5 Tralfamadoreans in the room. Write an expression for the probability that no two of them have the same birthday.
2. How would you find the smallest n for which a room of n Tralfamadoreans has probability at least $1/2$ of having two members with the same birthday?
3. The above two questions really require some sort of assumption to get an answer. In case you did not already provide one, what is the customary assumption one uses in probability exercises?
4. Write an expression for $\phi_X(t)$, the moment generating function of a random variable X . Find and interpret the second derivative $\phi_X''(0)$. If the MGF does not exist what would we use instead?
5. If X and Y are uncorrelated random variables must they be independent? If X and Y are independent random variables must they be uncorrelated? Explain in both cases.

6. For events A and B define $\Pr(A|B)$ in terms of $\Pr(A)$, $\Pr(B)$, $\Pr(A \cap B)$ and $\Pr(A \cup B)$. Write $\Pr(A | B)$ as the appropriate multiple of $\Pr(B | A)$.
7. When is $\Pr(A \cap B \cap C) = \Pr(A) \times \Pr(B) \times \Pr(C)$? You need not describe every sufficient condition, just one really good one.
8. State (a version of) Chebychev's inequality.
9. Write an expression for the variance of $X + Y$. Of course it is

$$\mathbb{E}((X + Y)^2) - \mathbb{E}(X + Y)^2,$$

but that is not the expression I want. Your expression should involve $\text{Var}(X)$ in a non-trivial way.

10. For what well-known distribution does the random variable X have $\Pr(X = x) = e^{-\lambda} \lambda^x / x!$ for integers $x = 0, 1, 2, \dots$ and a parameter $\lambda > 0$? What sort of quantity might be thought to have that distribution?
11. What is the probability density function of a normally distributed random variable with mean μ and variance σ^2 ?
12. Find the variance of a random variable X with the uniform distribution on $[0, 1]$, either by working it out or stating it if you remember the answer. (No looking it up on Wikipedia or elsewhere! Either know it or derive it or do both just to be sure.) Using the known answer for $U[0, 1]$, how would you work out the variance of the uniform distribution on $[-3, 3]$?
13. We have some random variables X_1, X_2, X_3 and so on. Suppose that $\Pr(X_i \leq x) \rightarrow F(x)$ as $i \rightarrow \infty$ for some CDF F . Does this mean that $\mathbb{E}(X_i) \rightarrow \mathbb{E}(X)$ where X is a random variable with distribution F ? If it does, either prove it, or state a well known theorem about it. If it does not, then come up with a counterexample.
14. The random variables $X_i \in \{0, 1\}$ are independent and identically distributed with $\Pr(X_i = 1) = p$ and $\Pr(X_i = 0) = 1 - p$. Their average is $\bar{X} = (1/n) \sum_{i=1}^n X_i$. What is $\text{Var}(\bar{X})$? Find the answer using whatever combination of memory and derivation works best for you.